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Estimating analytically the capacity of batch plants with shared equipment: a yoghurt plant case study

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Abstract

Estimates of maximum production capacity in food plants are required when planning, scheduling, debottlenecking or optimizing manufacturing efficiency. With the exception of very simple plants and process routings, analytical methods for calculating a plant's capacity are, in general, lacking. In this paper, a novel algorithm is presented for calculating analytically the minimum cycle time and capacity of batch processes with equipment shared across overlapping process steps. The algorithm explores alternative configurations with respect to the use of shared equipment across batches and selects the one that minimizes the cycle time. The implementation of the algorithm is demonstrated with the use of a yoghurt production process whereby the same vessels are used both for the fermentation step as well as the storage of the final product before feeding the filling machines. The optimal cycle time of the yoghurt process is determined by the algorithm and the corresponding maximum capacity is calculated.

Keywords: plant capacity; batch process; minimum cycle time; yoghurt process;

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1. Introduction

Plant capacity analysis is an important tool in the effort to improve manufacturing efficiency, maximize utilization of plant resources and eliminate process bottlenecks. The ability to efficiently plan and schedule production also depends on reliable estimates on plant production capacity.

Many plants in the food industry operate in batch or semi-continuous mode. In this mode, production consists of a series of sequential steps corresponding to a specific recipe or product routing. Each step has its own processing time and requirements for plant resources, the most important of which is the main equipment where this step is carried out. Calculating analytically production capacity of a batch plant is not, in general, an easy task considering that a typical food plant produces a variety of goods through different multi-step routings while using a multitude of, often shared, resources.

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An important concept in capacity analysis of batch or semi-continuous processes is that of cycle time defined as the time interval between the start of two consecutive batches. The smallest the cycle time, the more batches can fit within a given production period, and, therefore, the more product can be obtained at the end. More specifically, for a production period T (much longer than the cycle time of the process to minimize end effects), the production capacity P of a plant with batch size BS, can easily be calculated by the following equation:

$$P = \frac{T}{Ct} BS \tag{1}$$

where *Ct* is the implemented cycle time, i.e. the time interval between consecutive executions of batches. According to (1), maximum capacity can be realized by implementing the minimum cycle time so, for a given batch size, the question of maximum capacity is equivalent to the estimation of the minimum cycle time.

The derivation of the minimum cycle time is straightforward and well-established in the literature (see for example [1]) if all the steps in the batch process use distinct equipment pools and all equipment in the pool are identical. Let's consider a batch process with *n* steps each executed in a different single piece of equipment. If t_i is the duration of step *i*, then, the minimum cycle time of the overall process is equal to the maximum duration over all steps:

$$Ct_{\min} = \max_{i=1,..,n}(t_i) \tag{2}$$

The step with the maximum duration is the process *bottleneck* since it is the one that limits the plant capacity. If there exist multiple available equipment for each step (but there is no equipment sharing among steps), then the minimum cycle time of the process can be calculated as:

$$Ct_{\min} = \max_{i=1,..,n}\left(\frac{t_i}{m_i}\right) \tag{3}$$

where m_i is the number of equipment available for the execution of step *i*. For each step, available equipment can alternate from batch to batch effectively reducing the cycle time of that step. In the above analysis, all equipment in the pool are assumed identical; it follows that the step duration is independent of the equipment used, so t_i is a process step constant.

The estimation of the minimum cycle time becomes much more involved, however, if available equipment are not identical or different process steps share equipment from the same pool. Some equations and graphical tools for calculating plant capacity when equipment are non-uniform are presented in [2]. To the author's best knowledge, there are no analytical results in the literature for cycle time estimation for the shared equipment case. In these complex cases, estimates of plant capacity can be obtained only indirectly through optimal scheduling (see for example [3]). The need to obtain analytically estimates of the plant capacity in these complex cases is addressed in this paper with the help of the following example.

2. Motivating case study

A typical yoghurt process (modelled by a real plant) will be used as an example of a batch process with equipment sharing steps. The overall process consists of the following steps: pasteurization, fermentation, cooling, storage and filling. The Gantt chart in Figure 1 shows the sequencing in time of these steps in the corresponding equipment. We shall focus on the fermentation (in T1) and storage (in

T2) steps because these are the steps that share the same equipment pool; in addition, these are the known process bottlenecks with the longest durations (as it can be easily verified in Figure 1). Fermentation takes 12.9 hours and it involves the following subtasks: SIP (Steam In Place), receive the material from the pasteurizer, ferment for 8 hours, pump the material to the storage tank through the cooler and CIP (Clean In Place). Storage lasts for 4.73 hours and includes: SIP, receive the material from the fermentation tank through the cooler, feed the filling line and CIP. Within every batch, there is a 3.4 hours time overlap in the execution of the fermentation and storage steps due to their simultaneous use when the fermentation tank feeds the storage tank.

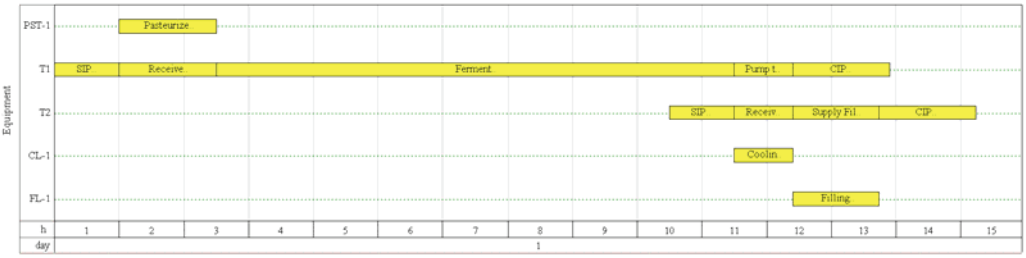

Fig. 1. Gantt chart of yogurt process

The plant has a total of 9 tanks that can be used either to ferment or store the fermented yoghurt; fermentation of a batch happens in any of the nine tanks and when it is over, the fermented yoghurt is cooled through a continuous-flow cooler and stored in another tank to feed the filling machines. Obviously, fermentation and storage within the same batch cannot use the same tank; so, this is a situation with two steps sharing the same equipment pool and also overlapping in time (sharing can happen across batches but not within the same batch). Motivated by that example, the analysis of cycle time in this paper will concentrate around batch processes with two overlapping steps sharing the same equipment pool. The objective is to find the minimum cycle time and, therefore, the capacity of the plant.

3. Methods

In the general case, let t_1 and t_2 denote the duration of the steps 1 and 2 sharing a pool of m equipment. Under the assumption that these steps are also the process bottlenecks, we will ignore in the analysis the remaining steps in the production recipe; in general, however, the contribution of all steps (sharing or not sharing equipment) to the process cycle time should be considered.

With two steps sharing the same equipment pool there exist two possible configurations with respect to the equipment's use: either equipment are divided into two groups with each group dedicated respectively to each step, or, all equipment are used by both steps in a rotating way.

In the first case of exclusive use, let m_1 and m_2 be the number of equipment dedicated to steps 1 and 2 respectively. Obviously, $m_1+m_2=m$. Since in this case the two steps have distinct equipment pools, Equation (3) applies and the minimum cycle time is now given as:

$$CT_{\min} = \max\left(\frac{t_1}{m_1}, \frac{t_2}{m_2}\right) \quad \text{where} \quad m_1 + m_2 = m \tag{4}$$

Different combinations of m_1 and m_2 yield different values for CT_{min}; therefore, all possible combinations should be explored to find the one that produces the smallest value of minimum cycle time. This would correspond to the optimal configuration for the exclusive equipment use case.

Things are considerably more complex in the rotating case. In this case, each piece of equipment is assigned consecutive executions of steps 1 and 2, so the sequence of steps executed in each piece of equipment will look like: 1-2-1-2-1-2 etc. Other configurations are possible (e.g. 1-1-2-2-1-1-2-2 etc.) but these do not correspond to cyclic scheduling at constant cycle time and will not be considered. In the assumed configuration, every execution of step-1 in any piece of equipment is followed by a step-2 that must belong to a later batch (due to the assumed time overlap between steps 1 and 2, these steps cannot be executed in the same equipment for the same batch) which, in turn, is followed by another step-1 that belongs to an even later batch (since other in-between batches could have started in the other available equipment.)

This situation is graphically depicted in the form of a Gantt chart in Figure 2. Three consecutive steps 1-2-1 (executed in any of the available equipment) are shown in Figure 2 by the aligned horizontal bars. The duration of each bar represents the duration of each step and the numbers within each bar represent the step-id and the batch-id respectively. More specifically, the leftmost bar is a step-1 executed as part of a batch which, without loss of generality, will be assumed to be the ‘reference’ batch with index 0. Following the rotation rule, a step-2 is executed next in that same equipment and the batch index that this step belongs to is denoted as K_2. K_2 must be a later batch than the reference, so $K_2>0$. Following again the step rotation, a step-1 will be executed next in the same equipment belonging to a batch with index denoted as K_1. Since, by construction, this is a later batch than the reference batch, it follows that $K_1>0$. K_1 must also be a later batch than K_2, since step-1 of K_2 (shown one line below in Figure 2) must have started even before step-2 which, by construction, is earlier than K_1. It follows that $K_1 > K_2 > 0$.

As indicated in Figure 2, let Z and W denote the time intervals between the three steps executed in the assumed equipment. For the schedule of these steps to be feasible (no overlaps between steps executed in the same equipment), Z and W must be non-negative. To derive expressions for Z and W and enforce their lower bound, we must consider the fact that, by definition, the time gap between the starts (or ends) of the same step in two different batches is an integer multiple of the cycle time. So, the time gap between the ends of step (1,0) and (1,K_1) is K_1Ct. Similarly for (1,0) and (1,K_2), the time gap in between their ends is K_2Ct. With this information and by inspecting Figure 2, the following equations (5) can be easily derived for Z and W:

$$\begin{aligned} Z &= K_2Ct - g \geq 0 \\ W &= (K_1 - K_2)Ct - (t_2 - g) - t_1 \geq 0 \end{aligned} \tag{5}$$

In the above equations, g represents the time overlap between step-1 and step-2 of the same batch (note in Figure 2 the overlap between (1,K_2) and (2,K_2) for batch K_2). This gap, g, is a constant parameter related to how the batch recipe is executed and can be considered known and independent of the schedule. Equations (5) can be solved for Ct and combined to yield the following lower bound on Ct:

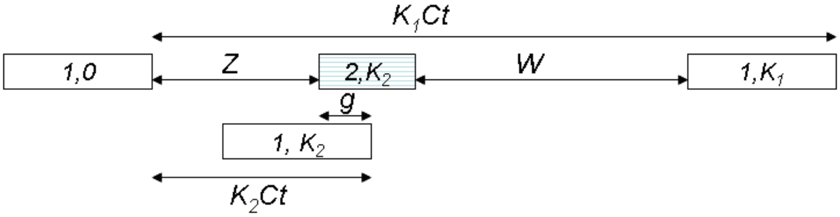

Fig. 2. Gantt Chart of a single equipment unit use by three consecutive steps 1-2-1

$$Ct \geq \max\left(\frac{g}{K_2}, \frac{t_1 + t_2 - g}{K_1 - K_2}\right) \tag{6}$$

The value of the second bound in (6) is minimized by the maximum value of K_1. Recalling that K_1 represents the batch index difference of two consecutive occurrences of step-1 in the same equipment, it can be easily deduced that the maximum value that K_1 can take is m, the number of available equipment. In other words, after at most m batches the execution of a step-1 will return to the same equipment. With this upper bound on the value of K_1, the minimum cycle time can be estimated from (6) as:

$$Ct_{\min} = \max\left(\frac{g}{K_2}, \frac{t_1 + t_2 - g}{m - K_2}\right) \quad \text{where} \quad 0 < K_2 < m \tag{7}$$

Equations (4) and (7) provide estimates of the achievable minimum cycle time for the two different equipment use configurations (exclusive equipment use and rotation). The optimal minimum cycle time and the corresponding configuration can be derived by exploring all possible values of m_1 and m_2 in (4) and of K_2 in (7) and choosing the one the yields the smallest value for Ct_{min}.

4. Results and Discussion

This calculation algorithm will be demonstrated for the yoghurt case study with the use of the process scheduling software SchedulePro™ (by Intelligen, Inc.) Using the established notation and the example data presented earlier, it follows that t_1=12.9 hours (the duration of step-1), t_2=4.73 hours (the duration of step-2), g=3.4 hours (the time overlap between the two steps) and m=9 (the number of available shared equipment). With these data and the use of Equation (4) the minimum cycle times for the exclusive equipment use case can be calculated for different combinations of m_1 and m_2. The results are shown in Table 1. The optimal cycle time of 2.15 hours corresponds to 6 tanks devoted to fermentation (m_1=6) and the remaining 3 tanks to storage (m_2=3). Since fermentation takes over twice as much time as storage, it should be expected that assigning twice as many tanks to fermentation should normalize their durations and yield the optimal policy. The optimal policy for the exclusive pool use case is shown in Figure 3 where the execution of 18 consecutive batches is presented in the form of a Gantt chart. Note that the fermentation step is the bottleneck since there is no slack in the use of the six dedicated tanks (T1-T6) between batches.

Table 1. Estimated minimum cycle time for the exclusive equipment use configuration

m_1	m_2	CT_{min} (hours)
1	8	12.9
2	7	6.45
3	6	4.3
4	5	3.225
5	4	2.58
6	**3**	**2.15**
7	2	2.365
8	1	4.73

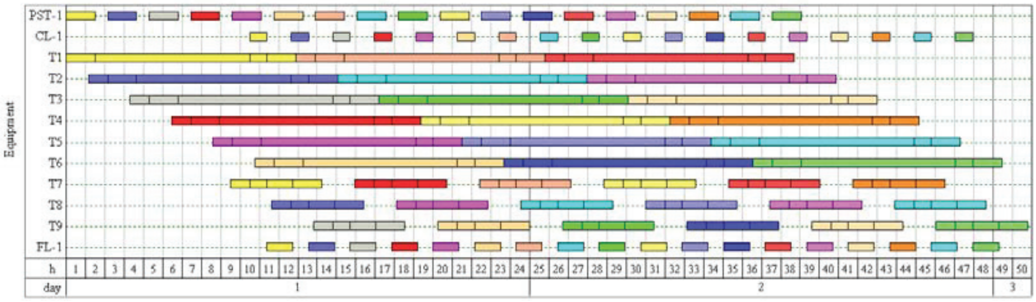

Fig. 3. Production schedule of the yoghurt plant for minimum cycle time in the exclusive equipment use case

Table 2 shows the results for the rotation case derived by Equation (7) with different values of K_2. The optimal cycle time of 2.033 hours is obtained for $K_2=2$ which means that the tightest schedule corresponds to the case where each tank alternates between fermentation and storage with the storage step belonging to the next to the following batch from the fermentation batch. This is depicted in Figure 4 where again a schedule of 18 consecutive batches spaced by the calculated optimal cycle time is presented. The block arrows in the chart demonstrate how the rotation works in the optimal case. A fermentation step is executed in T3 for the 'grey' batch; the next task that T3 undertakes is the execution of the storage step of the 'magenta' batch which is second in row after the 'grey' batch. Step-1 of the 'blue' batch follows in T3; the 'blue' batch is ninth in row after the 'grey' batch. Each step, therefore, returns to the same equipment every nine batches, a number corresponding to the total number of available vessels.

Table 2. Estimated minimum cycle times for the rotating equipment use configuration

K_2	CT_{min} **(hours)**
1	3.4
2	**2.033**
3	2.372
4	2.846
5	3.558
6	4.743
7	7.115
8	14.23

Note that the optimal cycle time in the rotation case (2.033 hours) is smaller than the cycle time in the exclusive use (2.15 hours) so, in this example, rotation represents the best possible configuration. This can also be verified by comparing Figures 3 and 4: in the rotation case 2 days are enough for the execution of the 18 batches, while this is not the case in the exclusive pool use case.

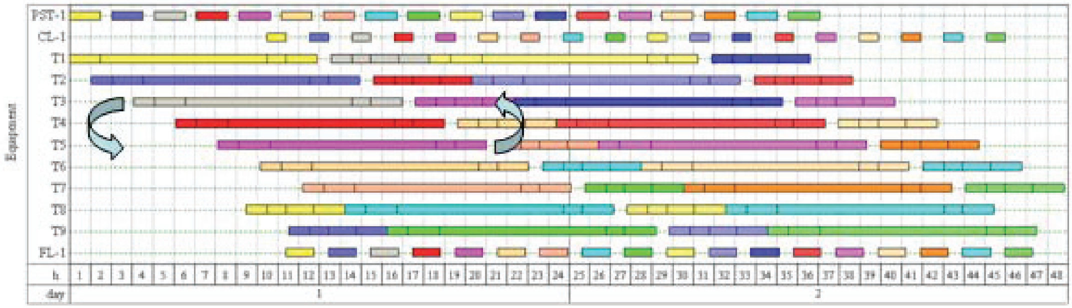

Fig. 4. Production schedule of the yoghurt plant for minimum cycle time in the equipment rotation case

With the minimum cycle time calculated previously over all possible configurations and an assumed batch size of 8 metric tons (MT), equation (1) can be used to calculate a maximum annual production capacity of 34471 MT (assuming 365-day, 24/7 operation). This amount corresponds to the execution of approximately 4309 (=365·24/2.033) batches per year. It should be stressed that all these calculations are dependent upon the selection of the batch size since the batch size affects (in a non-proportional way) the duration of the process steps. If the batch size is also a decision variable, the above analysis should be repeated for different values of batch size to determine the optimal plant capacity.

5. Conclusions

Analytical expressions for the minimum cycle time of complex batch processes are lacking. Even though it is doubtful whether the problem can be solved analytically for the general case, it is still manageable for particular sub-cases with practical interest such as the one presented in this paper. Minimum cycle time estimates can be used to determine the maximum capacity of a batch plant, identify the process or equipment bottleneck and schedule production efficiently, so the motivation for deriving analytically such estimates is significant both from a theoretical as well as a practical point of view.

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